

Introduction to Algebra

Magic Trick: Think of any number between 1 and 100. Multiply the number by 2, add 1 and multiply the answer by 5, add 4. Tell me the answer and I can tell you the number which you have in mind.

Q1 How can you find this answer? Can I guess the answer? If I guess your number, say 71, can I follow the steps- $71*2=142$, $142+1=143$, $143*5=715$, $715+4=719$ so that I cross check my answer, which in this case is 719 with your answer- my guess is correct in case my number (719 in this example) matches with your answer, can I try this method? Should I try this with all numbers from 1 to 100?

Q2 Why does the final answer look like your number followed by 9? [for example if you were to take 72, the final answer would be 729]. Does this work only for 71 and 72 or does this work for all numbers? Would you want to try out some more numbers?

Q3 Observe the process for 71- $2*71$, $(2*71)+1$, $5*((2*71)+1)=((10*71)+5)$, $((10*71)+5)+4=((10*71)+9)$
Now let us replace the number 71 with Rajnikanth.

Step1: $\text{Rajnikanth} * 2$

Step2: $(\text{Rajnikanth} * 2) + 1$

Step3: $5 * ((\text{Rajnikanth} * 2) + 1) = 10 * \text{Rajnikanth} + 5$

Step4: $((10 * \text{Rajnikanth}) + 5) + 4 = (10 * \text{Rajnikanth}) + 9$

Does this prove the observation in step2? What is the place value of Rajnikanth? What is the unit's place of the expression $((10 * \text{Rajnikanth}) + 9)$?

Q4 We wrote $5 * ((2 * 71) + 1) = 10 * 71 + 5$ by using distributive law- that $a * (b + c) = (a * b) + (a * c)$, but then does this rule also apply to Rajnikanth? Rajnikanth is nothing but your number right, so if this rule applies to all natural number, will not apply to Rajnikanth as well?

Q5 Why did we replace the number 71 with Rajnikanth? The number 71 talks of one random observation, but what about the expression with Rajnikanth? Does the observation in step4 apply to all natural numbers or does it apply to just 71? Did we come up with a proof that no matter what number we take, we'd get that number followed by 9 as an answer?

So now that you saw the power of Rajnikanth, let us study Algebra- where numbers are replaced by alphabets. If we don't know the answer, but if we have some information around the answer, we could just use an alphabet and do math with it. Even though we don't know the number, we could follow a series of steps with the alphabets and then find out our answer.

Short Quiz

Q1 What values can "Rajnikanth" take?

Q2 Can we find "Rajnikanth" if we are told that "Rajnikanth" + 2 = 5?

Q3 Should "Rajnikanth" even take a value?

Q4 Can you Identify the properties followed by "Rajnikanth"- associative, distributive, commutative?

Q5 Is "Rajnikanth" a mathematical object? What do you know about "Rajnikanth"?

Exercise 1

Q1 There are 9 rows of cadets and we have 6 cadets in each row. How many cadets are marching? Can the answer be $9 + 9 + 9 + 9 + 9 + 9 + 9$? Can the answer be $6 + 6 + 6 + 6 + 6 + 6 + 6 + 6 + 6$? What is repeated addition? Is $7 \times 9 = 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7 + 7$ and is $7 \times 9 = 9 + 9 + 9 + 9 + 9 + 9 + 9$?

Q2 If the price of one chocolate is Rs10 and the price of one ball is Rs20, what is the total amount payable to the shopkeeper? Why are you using addition here?

Q3 We don't know the number of cadets, so we write down 'n' as the number of cadets and we have 6 cadets in each row. How many cadets are marching?

Q4 Let number of rows be 'n' and number of cadets be 's', how many cadets are marching?

Q5 If Raju buys mangoes for Rs x and bananas for Rs y, what is the total amount of money which he spent? Not sure what to do? What if the price of mangoes is Rs50 and the cost of bananas is Rs20? What operation are you using and why? So if you were put x in place of 50 and y in place of 20 what do you get? Is this similar to the substitution of "Rajnikanth" in place of 71?

Q6 If you purchase 'x' mangoes for Rs20 each and 'y' bananas for Rs5 each, what is the total amount of money which you would have paid? If the total number of mangoes was 10 and total number of bananas was 5, what would be the answer? Can you now plug in 'x' in place of 10 and 'y' in place of 5?

Q7 If you have purchased 'x' mangoes for Rs 'p' and 'y' mangoes for Rs 'q', what is the total amount which you would have paid the shopkeeper? [Can you try the toy-example method here?]

Q8 If you have purchased 'x' mangoes for Rs 'p' and 1 orange for Rs q and the shopkeeper bills you Rs100, what can you comment about 'x', 'p' and 'q'?

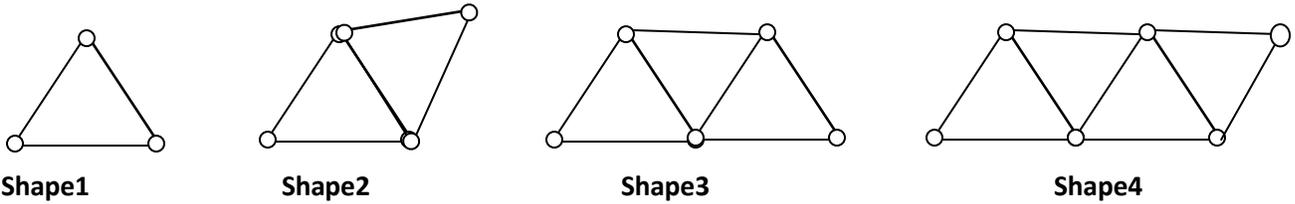
Q9 Radha's age is 5 years lesser than Ravi's age. Radha's age is 'x' years while Ravi's age is 'y' years. Can you find a relation between 'x' and 'y'? (Note that in mathematics, information is expressed in the form of an equation. If you don't get the answer, consider a toy example- values for x and y, to see a pattern)

Q10 Chocolates are taken out of a large box and transferred into three small boxes. However, 10 chocolates were left out. If the number of chocolates in the large box was 'x' and if the number of chocolates in small box was 'y', express the relationship between 'x' and 'y' through an equation.

Toy Example Technique

- You learn to ride a toy-cycle and then learn to ride a bicycle, isn't it? Don't toys help?
- If the problem is hard to understand, change the letters to numbers, understand the logic.
- Once you understand the logic, check if the logic can be extended to numbers.
- Be careful while assigning numbers. For instance if they tell us that Ravi's weight is 5kg more than Raju's weight, we can't assign 50 and 60 as the values of their weights. Why? Are these values consistent with the constraint that their weights differ by 5kg?
- For example, if they ask us to find the class average, given that the students marks are x,y,z, then we'd substitute values in place of x,y,z- may be 100, 95 and 90, we get the class average as $(100 + 95 + 90) / 3 = 95$. So class average = Total marks/ Total number of students = $(x + y + z) / 3$

Tougher Question: Observe the following pattern. Develop a formula to find out the number of match sticks in the n^{th} object.



Q1 Do you see a pattern? Can you draw the next shape in the pattern (Shape5)? How many triangles are used to make shape1? How about shape 2,3 and 4? How many circles or lines do you see in each shape? Do you see any pattern?

Q2 Your friend Adil gives you a wrong formula – $3 * n$ (number of match sticks required to make shape n is equal to $2 * n$). How do you prove that his answer is wrong? Does his formula work for $n = 1$? Does it work for $n = 2$? For $n = 3$? (at $n = 1$, does shape n have $3 * n$ number of match sticks?)

Q3 How many match sticks do you see in shape1 (call the answer A_1)? How about shape2 (call the answer A_2)? How about shape3(A_3)? How about Shape 4(A_4)? Do you see any pattern? What is the relation about A_1 and 1, A_2 and 2, A_3 and 3, A_4 and 4? Is $3 * n$ is a wrong formula, what can be the correct formula?

Q4 How many match sticks do you need to build shape1? How many extra sticks are needed to transform shape1 into shape2? How many extra sticks are needed to change shape2 into shape3? How about the number for changing shape3 into shape4? So how many sticks get added in each step? How many sticks are needed to change shape1 into shape3? How many sticks are needed to change shape1 into shape4? How about the required number, for changing shape1 into shape5?

Q5 You are required to build shape10. Let us assume that we will build it step by step, we build shape1, shape2 and so on. How many sticks do we use to build shape1? How many extra sticks do we add for shape2? For shape3? For shape 4? How many times do you add sticks and how many sticks do you add? So how many sticks do you use in shape 10? Do you see a pattern, can you use this toy example to find the answer for shape n ?

More patterns (Find the nth term)

Q1 4,6,8,10,12....

Q2 3,9,12,15,18...

Q3 4,10,13,16,19...

Q4 1,4,9,16,25,36...

Q5 3,7,12,19,28,39...

Hints: Play with all possibilities- check the difference between terms- multiples, squares, cubes etc.