Division of Polynomials

An expression of the form $\sum_{i=0}^{n} a_i \; x^i$= $a_0 + a_1x^1 + a_2x^2 + a_3x^3 + a_4x^4 \ldots a_nx^n$ is called a polynomial

Short Quiz: Consider the polynomial p(x)= $1 + 2x + 5x^2 - 8x^7$ [15 minutes]

Q1 What is the largest power of x? (This is called the degree)

Q2 What is the number with which x^2 is being multiplied with, in this polynomial (called coefficient)?

Q3 Is f(x)= 10 a polynomial? (Can it be expressed as $\sum_{i=0}^{n} a_i \; x^i$? If yes state all a_i) What is its degree?

Q4 What is the value of p(x) when you substitute x=2? [Written as p(2)]

Q5 What is the value of p(1)? Is it zero? (a such that p(a)=0 is called a zero of the polynomial)

Q6 What are the zeroes of p(x)=x^2-x?

Q7 Is p(1) equal to sum of all its coefficients?

Q8 Is the above statement true for one polynomial or all polynomials?

Q9 Can the degree of a polynomial be equal to 3.5? Can a coefficient be equal to 3.5?

Q10 Is p(1)+p(-1) an even number? Why?

Polynomial and Place Values Quiz [15 minutes]

Let us write a corresponding polynomial $p_M(x)$ for a number M=$m_n m_{n-1} \ldots m_3 m_2 m_1$ as p(x)= $\sum_{i=1}^{n} m_i \; x^i$. If

Q1 What is the value of $p_M(0)$? What about $p_M(1)$?

Q2 Can you write down the polynomial $p_M(x)$ if M=4589?

Q3 What is the value of $p_M(10)$?

Q4 $p_M(1)$ is divisible by 9, what can you comment?

Q5 $p_M(-1)$ is divisible by 11, what can you comment?

Now consider the polynomial $q_M(x)$= $\sum_{i=1}^{n} m_i \; x^{i-1}$ [20 minutes]

Q1 If M=4589, can you write down $q_M(x)$?

Q2 What is the degree of $q_M(x)$? (for any M= $m_n m_{n-1} \ldots m_3 m_2 m_1$)?

Q3 What can you comment if $q_M(0)$ is divisible by 2?

Q4 Can you think of a divisibility test for 5,10?

Q5 What happens to $q_M(x)$ as x gets close to -∞ if n is even? What if n is odd?

Division of Algebraic Expressions [20minutes]

Q1 Is 12 divisible by 6? Why do you say so? Likewise is x^2 divisible by x?

Q2 Is 15 divisible by 6? But then 15 = 2.5* 6?

Q3 Is x^3 divisible by x^4? But then isn't $x^3 = x^4 * \frac{1}{x}$? Is $\frac{1}{x}$ a polynomial? Does it matter?

Q4 When is a natural number a divisible by a natural number b?

Q5 When is a polynomial p(x) divisible by another polynomial q(x)?

Q6 What is the divisor when x^3+3x^2+2x is divided by x^2+3x+2?

Q7 How do you divide 15 by 6? Do you get a remainder? So can you divide any natural number by any other natural number?

Q8 What is the divisor when you divide x^2+3x by x+2? What is the remainder?

Q9 Can the remainder exceed the divisor? Can you get a remainder 5 when you divide by 4? Why not?

Q10 Can the degree of remainder polynomial exceed the degree of divisor polynomial? Why not?

Long Division Method: Questions for discussion [20minutes]

Q1 How do you divide 22356 by 134? Why do you use long division method?

Q2 Can we try long division method to divide $p_{22356}(x)$ with $p_{134}(x)$? What do you think?

Q3 Let q(x) be the divisor when $p_{223456}(x)$ is divided by $p_{134}(x)$. What is the degree of q(x)? Why?

Q4 What is the coefficient of x^3 in q(x)? Why?

Q5 What is the coefficient of x^2 in q(x)? Why? Does this remind you of long division method?

$p_{223456}(x) = 6x+5x^2+4x^3+3x^4+2x^5 +2x^6$ and $p_{134}(x)= 4x + 3x^2+x^3$.

Step1: Division is nothing but repeated subtraction. So can we subtract multiples of $p_{134}(x)$ from $p_{223456}(x)$? What if I subtract $2x^3p_{234}(x)$? What is the degree of $p_{223456}(x)-2x^2*p_{134}(x)$?

Well $2x^3* p_{134}(x) = 8x^4 + 6x^5 + 2x^6$

So $p_{223456}(x)-2x^3*p_{134}(x) = (6x+5x^2+4x^3+3x^4+2x^5 +2x^6) – (8x^4 + 6x^5 + 2x^6) = (-4x^5-5x^4 +4x^3+5x^2+6x)$

Step2: So after one subtraction, we are left with $(-4x^5-5x^4 +4x^3+5x^2+6x)$

Can we now subtract this with $-4x^2 *p_{134}(x)$?

So $(-4x^5-5x^4 +4x^3+5x^2+6x)-(-4x^2*p_{134}(x)) = (-4x^5-5x^4 +4x^3+5x^2+6x)+(4x^2*p_{134}(x))$

$= (-4x^5-5x^4 +4x^3+5x^2+6x)+(16x^3+12x^4+4x^5)$

$= (7x^4 +20x^3 + 5x^2+6x)$

Step3: Can we now subtract $7x*p_{134}(x)$ from $(7x^4 +20x^3 + 5x^2+6x)$

$(7x^4 +20x^3 + 5x^2+6x) – 7x*p_{134}(x) = (7x^4 +20x^3 + 5x^2+6x) – (28x^2 + 21x^3 + 7x^4)$

$= (-x^3 – 23x^2 +6x)$

Step4: Can we now subtract $-1*p_{134}(x)$ from $(-x^3 – 23x^2 +6x)$

$(-x^3 – 23x^2 +6x)- (-1*p134(x))= (-x^3 – 23x^2 +6x)+p_{134}(x) =(-x^3 – 23x^2 +6x) + (4x + 3x^2+ x3)= (-20x^2+10x)$

Question: If the degree of the remainder polynomial is more than that of divisor polynomial, can you do atleast one more subtraction? Does that mean by you didn't complete the repeated subtraction process? For instance if divisor is $2x^2+3x$ and if the remainder is x^3+x^2+x+1, can you perform one more subtraction? With what multiples of the divisor would you perform the subtraction?

Find the quotient and remainder if the Dividend polynomial D(x) and divisor polynomial d(x) are

Q1 $D(x)= 5x^2 + 3x +1$ $d(x)=x + 5$

Q2 $D(x) = 6x^4 + 3x^3 + 2x^2 + 1$ $d(x)=x+6$

Q3 $D(x) = 6x^4 + 3x^3 + 2x^2 + 1$ $d(x)=3x^2 + x+6$

Q4 $D(x) = 6x^4 - 3x^3 - 2x^2 + 1$ $d(x)=3x^2+ 2x-6$

Q5 $D(x) = 9x^5 - 6x^4 + 5x^3 - 2x^2 + 1$ $d(x)=3x^3-2x^2+x+6$

Commentary

1- Algebra has a lot of notation, if that's confusing translate the question/argument to English

2- If an algebraic argument isn't understood, try and use an example from arithematic. For example while understanding why degree of the divisor polynomial is more than degree of remainder polynomial, we looked at division of numbers- why remainder can't be 5 if you're dividing by 4!

3- Steps of long division method can be understood as repeated subtraction

4- Do you now understand why Ramanujan the Number Theorist played with polynomials?

5- We haven't memorized the techniques, we chose to find them out ourselves, so that we don't forget them or even if we do, we can recall it with ease as we know the logic used to derive them.